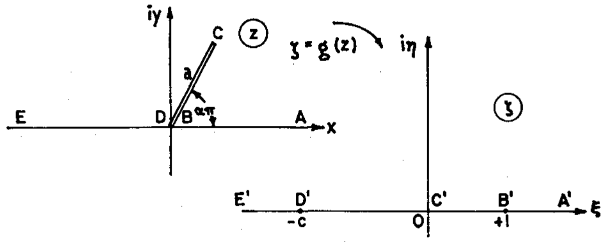

Fig. 5 Physical and mapped half-planes, V-wing.

Table 1. Dimensionless added mass of three shapes vs dihedral angle.

β deg.	M/ρπa²		
0	1.000	1.000	1.000
5	1.015	.992	.960
10	1.017	.974	.908
15	1.005	.944	.846
20	.981	.903	.777
25	.945	.852	.701
30	.896	.792	.621
35	.838	.725	.539
40	.771	.652	.457
45	.697	.576	.378
50	.617	.497	.303
55	.533	.418	.234
60	.448	.341	.173
65	.363	.267	.119
70	.280	.198	.076
75	.201	.135	.042
80	.127	.080	.018
85	.060	.035	.004
90	0.	0.	0.

In case only the exterior fluid is to be counted (5) gives $M' = \rho(2\pi\alpha - a^2 \sin 2\alpha\pi)$. This is also given in Table 1 and Fig. 4.

V-Wing

In the case of straight half-wings with dihedral angle β the proper mapping is (Fig. 5)

$$dz/d\zeta = (\zeta + c)^{-\alpha}\zeta(\zeta - 1)^{-(1-\alpha)}$$

We have again chosen C' at $\zeta = 0$ and B' at $\zeta = 1$, and left a unknown; but now the location of D' is also unknown. For large ζ we have

$$\begin{aligned} dz/d\zeta &= (1 - \zeta^{-1})^{-(1-\alpha)}(1\,c + \zeta^{-1})^{-\alpha} \\ &\sim 1 + [(1-\alpha) - \alpha c]\zeta^{-1} \\ &\quad + \tfrac{1}{2}[(2-\alpha)(1-\alpha) - 2c\alpha(1-\alpha) + c^2\alpha(1+\alpha)]\zeta^{-2} + \\ &\quad\quad 0(\zeta^{-3}) \end{aligned}$$

Now it is clear that the coefficient of ζ^{-1} must vanish to avoid a log ζ term in the mapping (7): thus $c = (1-\alpha)/\alpha$. The coefficient of ζ^{-2} then reduces to $A_1 = (1-\alpha)/2\alpha$, so $M = \pi\rho(1-\alpha)/\alpha$. As before, a is found as the definite integral

$$a(\alpha) = \int_0^1 (1 - \xi)^{-(1-\alpha)}(c + \xi)^{-\alpha}\xi \, d\xi$$

In this case the integral could not be reduced to a tabulated form, so it was evaluated numerically. The dimensionless added mass $M/\rho \cdot \pi a^2$ is given in Table 1 and Figure 4 as a function of the angle $\beta = (\pi/2) - \alpha\pi$.

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Lift on Airfoils with Separated Boundary Layers

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THIS Note contains the salient features of a method for calculating the sectional lift coefficient c_l on an airfoil as a function of its angle of attack α and freestream Reynolds number Re_∞ $(= V_\infty c/\nu_\infty)$ even at large angles of attack beyond the maximum c_l. The details of the method are contained in Ref. 1 which also contains the computer program.

The theory proceeds as follows. An angle-of-attack α, a lift coefficient c_l, and a freestream velocity V_∞ are assumed and the Theodorsen method[2] is used to locate the forward stagnation point and the inviscid flow over the body. A boundary-layer analysis starting at the forward stagnation point and proceeding downstream along the upper and lower surface is then performed. The initial flow is laminar and then may become turbulent. The boundary layer is analyzed by using the Cebeci, Smith[3] finite-difference method in both the laminar and the turbulent regions. In the turbulent region, the boundary-layer equations are expressed in terms of an eddy-viscosity coefficient while in the laminar region the eddy-viscosity coefficient is set equal to zero. Transition from laminar to turbulent flow is based on a momentum Reynolds number of 640 for a favorable pressure gradient and of 320 for an unfavorable pressure gradient. Included in the transition criteria (should they be needed) are experimental relations proposed by Gaster[4] for the bursting of short laminar separation bubbles.

The boundary-layer calculations are carried downstream until separation, characterized by a zero shear stress at the surface, results. At the point of zero shear stress (the separation point) the pressure coefficient c_p is known from the Theodorsen inviscid analysis. The pressure coefficients at separation on the upper and lower surfaces are then plotted against the assumed c_l (Fig. 1).

The calculations (inviscid plus boundary layer) are repeated for other assumed c_l keeping α and Re_∞ constant, until the

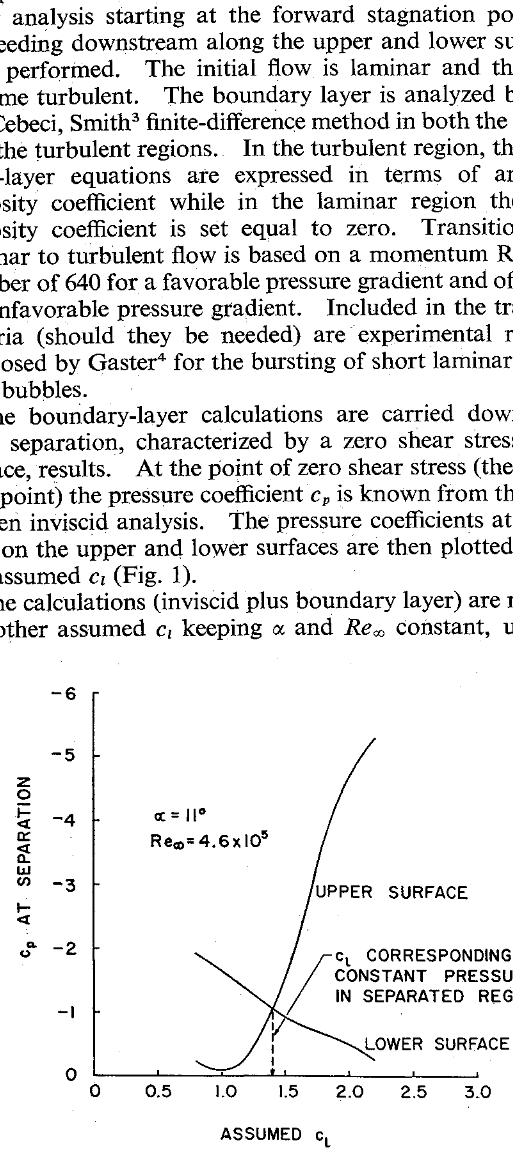

Fig. 1 Determination of c_l for prescribed values of α and Re_∞.

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line joining the pressure coefficients at separation for the upper surface crosses the line joining the pressure coefficients at separation for the lower surface (Fig. 1). The crossing point represents the correct c_l for the assumed α and Re_∞ as it satisfies the Thwaites' condition[5] that the pressure is constant in the separated region. The procedure is repeated for other α and Re_∞ and the theoretical curves in Fig. 2 result.

The theory was compared to test data obtained on the cambered elliptical wing section shown in Fig. 2. This wing, whose span is 16.5 in., has a thickness-chord ratio of 20%, a 5% camber, a chord $c = 8.6$ in., and a circular trailing edge of $\frac{1}{2}$ in. radius; 34 pressure taps are arranged around its periphery at the wing midspan. The tests were run in the West Virginia University subsonic tunnel at geometric angles-of-attack $\alpha_g = 0°, 5°, 10°, 12°$ and a nominal tunnel velocity $V_g = 105$ fps. The geometric angles of attack of the tests were corrected for the downwash w by the method of Ref. 6 which is improved still further in Ref. 1 to include high values of α_g. These tests were part of a continuing program to study the feasibility of obtaining high-lift devices by blowing around bluff-ended bodies.[7,8] The present tests with no-blowing ($V_J = 0$, Fig. 2) are needed to give the lower limit of the effectiveness of jet blowing on the sectional lift coefficient c_l.

It is noted that the theory compares favorably with the test data even at high values of α. Although the analysis in the present instance has been applied to a bluff-ended body it should apply, equally as well, to airfoil sections with pointed trailing edges.

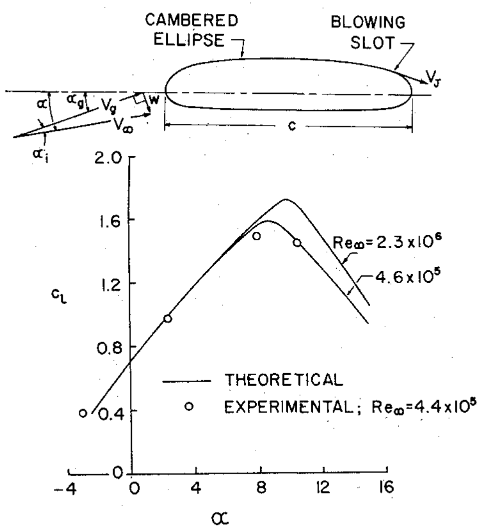

Fig. 2 c_l **vs** α**; comparison of theoretical curves with experimental data.**

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